SIMPLE HARMONIC MOTION

OBJECTIVE:

Investigating the concept of the spring constant *k* as a measure of the stiffness of the spring.

IDEA TO REMEMBER!

Springs have a constant stiffness, called *k*!

MATERIALS:

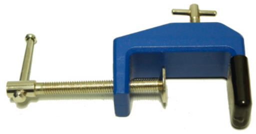


Table Clamp for Rod

Stopwatch

Digital Scale

PASCO 550 Interface

Hooked Weights

Pendulum Clamp

Meter Stick

Rod

Spring

PASCO Motion Sensor

CONCEPT:

THINK: What do you imagine when you read "Harmonic Motion"?

In a nutshell, it describes any repeating, back and forth oscillation over a consistent period of time. It is easily recognizable because it creates a sinusoidal curve when plotting distance vs time. Engineers love things that produce this curve because it is easy to predict their behavior and then develop technology around it. Indeed, this concept has been applied in numerous technologies and can be seen in natural life, too. See the *Real World Applications* section below for examples! To observe this in our lab we will use springs, which provide a useful model of this oscillatory behavior.

In 1660 (360+ years ago!), Robert Hooke developed his famous Hooke's Law, which describes the "restoring force" observed when a stretched or compressed spring is released, see Figure (1). Hooke found that this force is

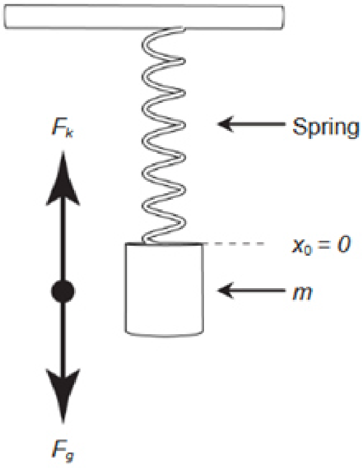


Figure 1

proportional to the total distance traveled, Δy, from equilibrium to the max stretched or compressed position. Thus, *a 25 year old* gave us the first step to understanding simple harmonic motion. Not bad!

IDEA TO REMEMBER!

Springs have a constant stiffness, called k!

If a mass m is attached to one end of a hanging spring, Hooke's spring force is equal to the gravitational force (Newton's second law) at the equilibrium position:

$$F = -k\Delta y = ma \tag{1}$$

where k is the stiffness constant. Realize that **the motion around a circle can be plotted across time to create a sine wave, see Figure (2)**. This means we can use angular frequency ω to calculate the frequency or velocity of an oscillation:

$$\omega = \frac{\Delta\theta}{\Delta t} = 2\pi f = \frac{2\pi}{T} \tag{2}$$

Since ω is part of the harmonic motion equations, we can solve Equation (1) for acceleration a and derive:

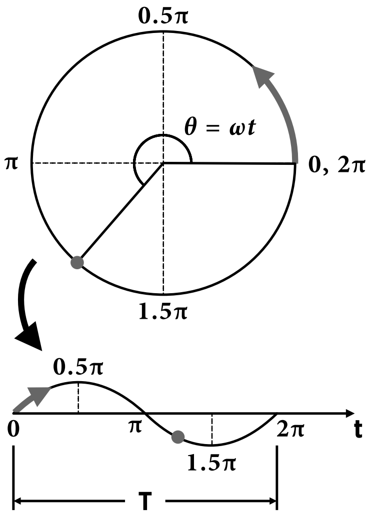
$$\omega = \sqrt{\frac{k}{m}} \quad (3) \qquad T = 2\pi\sqrt{\frac{m}{k}} \quad (4)$$

Now, let's also consider a fraction of the mass of the spring fm_s in the oscillation to improve this equation as:

$$T = 2\pi\sqrt{\frac{m + fm_s}{k}} \tag{5}$$

$$T^2 = \left(\frac{4\pi^2}{k}\right)m + \left(\frac{4\pi^2 fm_s}{k}\right) \tag{6}$$

Notice anything familiar about Equation (6)? Compare that to a linear equation ($y = mx + b$). The plot of T^2 vs m has a linear relationship with stiffness! Therefore, **the period of the system is related to how stiff the oscillator is**.



π is a universal constant; no matter the size curve, it can be a universal position "marker" along any periodic curve. Since the sine curve is periodic, we can use π to derive the change in position and the period T for angular velocity/frequency ω.

Figure 2

x, y: m, T^2

$$\text{Slope} = \left(\frac{4\pi^2}{k}\right) \text{ and y-intercept} = \left(\frac{4\pi^2 fm_s}{k}\right)$$

However, since the acceleration is not constant and depends on the stretched or compressed position of the spring, we will look deeper into the Equations above and apply more advanced techniques to predict more about the behavior. Solving Equation (1) for acceleration a in its derivative form we get:

$$\frac{d^2y}{dt^2}m = -ky$$

The function for position from this second-order derivative is:

$$\frac{d^2y}{dt^2} \Rightarrow y(t) = c_1 sin\theta + c_2 cos\theta$$

Where c_1 is the initial position at t = 0 and c_2 is the initial speed. We need to convert circular angle θ to radians because the curve repeats every multiple of 2π, so we can relate the equation to our system by

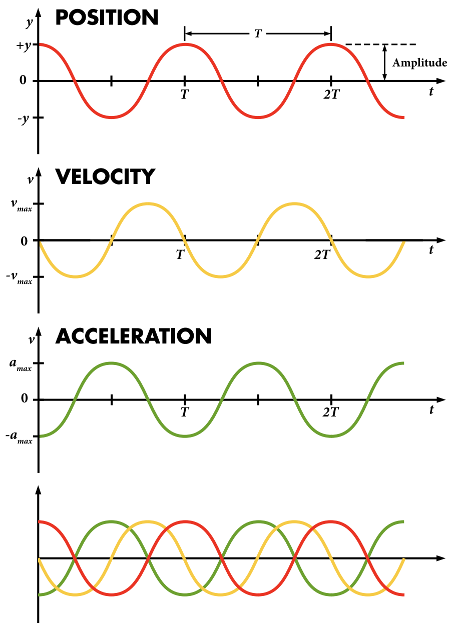


Figure 3

using the product of angular velocity and time ωt to give us the position. In practice, we can drop one of the θ functions (sine or cosine) with a phase shift ϕ, and then we can add a y-axis multiplier (A) to get the final equation for position with respect to time:

$$y(t) = Asin\left(\omega t + \phi\right) \tag{7}$$

Notice, ω can be substituted with Equation (2) or (3):

$$y(t) = Asin\left(\sqrt{\frac{k}{m}}t + \phi\right) \;\; OR \;\; y(t) = Asin\left(\frac{2\pi}{T}t + \phi\right) \tag{8}$$

Differentiating the position equation gives us the function for velocity over time, and differentiating the velocity function gives us the equation to model acceleration over time:

$$v(t) = A\omega cos\left(\omega t + \phi\right) \tag{9}$$

$$a(t) = -A\omega^2 sin\left(\omega t + \phi\right) \tag{10}$$

You can visualize these functions in Figure (3).

IDEA TO REMEMBER!

Springs have a constant stiffness, called k!

Real World Applications

- Harmonic oscillations are *everywhere!* They are the source of virtually all sinusoidal vibrations and waves, but it is very difficult to find a perfect simple harmonic oscillator in nature; most oscillators are *damped*, meaning something else is influencing the displacement of whatever the “spring” is.
- Some examples of harmonic oscillators are a bouncing ball, the shock absorbers in your car, electronic circuit clocks, stringed musical instruments, the pendulum motion of a play swing, the tides of the oceans and lakes, and even planetary motion.
- The harmonic oscillator potential can be used as a model to approximate many physical phenomena.

1) Combining Fourier math and harmonic motions to create awesome!

2) Quantum springs!

PRECAUTIONS:

A falling mass can cause injury. *Be cautious of things under the mass and only pull on the spring–mass in the manner described below!*

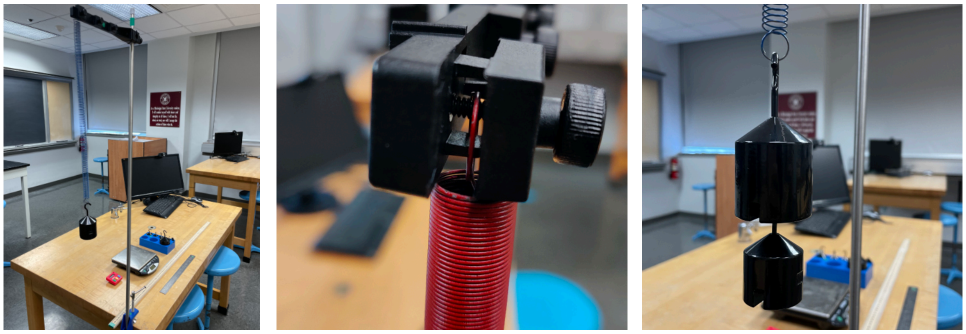
PROCEDURE:

Part 1

1. ☐ Fill out the top information on the worksheet.

2. ☐ REQUIRED: Read the *Concept* section.

3. ☐ Assemble the setup as shown in Figure (4).
    3.1. Make sure the spring is screwed tight under the pendulum clamp, see Figure (4b).
    3.2. Pick a mass between 200g and 800g (overall you need to try it for six different masses) and hang it on the spring. You can couple the mass as shown in Figure (4c).



(a) (b) (c)

Figure 4

4. ☐ Record the color and mass of your spring on your worksheet under "Part 1."

5. ☐ Pull the mass down 3cm *<u>very precisely</u>* using the meter stick and let the mass start oscillating.

6. ☐ While the mass is oscillating and at the *<u>exact</u>* moment the mass reaches the bottom of its travel, start timing the oscillation for 10 full periods.

7. ☐ Now that you have the overall time for the 20 oscillations, divide the overall time by the number of the oscillations to find the Period T (s) for each oscillation.

8. ☐ Repeat steps 2-5 for four different masses and fill out Table 1 on the worksheet.

9. ☐ Answer Questions 1 on the worksheet.

IDEA TO REMEMBER!

Springs have a constant stiffness, called *k*!

CONCEPT & PROCEDURE VIDEOS:

Part 2

1. ☐ Measure the initial length of the spring <u>without</u> any mass attached.

2. ☐ Hang one of your selected masses on the spring and measure the length of the spring.

3. ☐ Fill out the mass m (kg) and displacement y (m) on Table 2 on the worksheet. Remember that displacement is the difference of the stretched length minus the initial length.

$$v(t) = A\omega cos\,(\omega t + \phi)$$
$$v(t) = A\omega cos\,(\omega t + \phi)$$

4. ☐ Use Newton's second law of motion to calculate the force F (N) on the spring for Table 2.

5. ☐ Repeat step 2-4 for all four masses you selected in Part 1.

6. ☐ Plot F (N) vs Δy (m) using Excel, graph paper, or the back of your worksheet, and ask your TA to sign off on your worksheet that they saw your plot.

7. ☐ Answer Questions 2 & 3 on the worksheet.

IDEA TO REMEMBER!

Springs have a constant stiffness, called *k*!

Part 3

Imagine you are an engineer designing a compact machine with the exact spring system in this lab. You are trying to determine the position of the 0.5kg mass at a certain time range to keep other moving parts from colliding. In this part, we will show you how to use the harmonic motion equations based on your physical measurements from Part 2 and verify it with a motion sensor.

1. ☐ Open Capstone on the computer and connect the motion sensor to the PASCO interface.

2. ☐ Double-click on Graph on the right, then select "Position" for the y-axis to plot position vs. time.

3. ☐ Place the sensor under the hanging mass as shown in Figure (5).

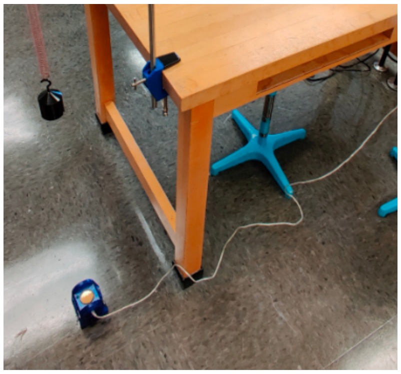


Figure 5

4. ☐ Suspend 0.5kg on the spring and make sure the distance from the bottom of the mass when it is at equilibrium to the sensor is around 30cm.

5. ☐ Pull the mass down 3cm very precisely using the meter stick and let the mass start oscillating.

6. ☐ While the mass is oscillating and at the exact moment the mass reaches the bottom of its travel, start the motion sensor recording. (The *Record* button is near the bottom-left under the graph area.) Take the data for at least 10 full oscillations.

IDEA TO REMEMBER!

Springs have a constant stiffness, called *k*!

7. ☐ Click *Stop* to finish recording. There should now be a sinusoidal curve on the graph.

8. ☐ Find the Period *T* of the wave using the *Coordinate tool* to get the values of the peaks. You may need to increase the decimal places of the tool: right-click > *Tool Properties* > *Numerical Format* > *Vertical Coordinate* > increase the value of *Number of decimal places*.

9. ☐ Answer Question 4 on the worksheet.

10. ☐ Use the *sine fit* tool to verify your equation variables.

11. ☐ Answer Question 5 on the worksheet and follow the **Let's THINK!** instructions below.

Let's THINK!

- **Ask questions**: What are you learning here?… Why is this Physics concept important and how can it be used?… What do you not understand?… (For more information on this Physics topic, scan the QR codes in the *Real World Applications* and at the start of the *Procedure* section.)
- **Discuss** the concept and demonstration with your partner to help each other understand better. Discussion makes learning active instead of passive!
- For **FULL PARTICIPATION [15 points]** you must call on the TA when you have finished your group discussion to answer some comprehensive questions. If you do not fully understand and the TA asks you to discuss more, you must call on them one more time to be dismissed with full marks.
- **CONCLUSION [10 points]**: In the Conclusion section at the end of the worksheet, write 3 or more sentences summarizing this concept, how this lab helped you understand the concept better, and the real world implications you see. Do you still have questions? If so, write those as well.

Updated Date	Personnel	Notes
2022.08	Chase Boone, Ahmad Sohani, Bernard Osei, Brooks Olree	2022 Summer Improvement: Created new format.
2022.08, 2023.01	Chase Boone	Corrections and clarifications.

Name: ______________________________ *PH2233* Section #: __________

Name: ______________________________ TA Name: ____________________

SIMPLE HARMONIC MOTION

WORKSHEET [70 points]

IDEA TO REMEMBER!

Springs have a constant stiffness, called *k*!

Part 1

Color of Spring: ____________ [1 point]

Mass of Spring: ____________kg [1 point]

Table 1: Squared Period of Oscillation vs Mass [8 points]

Mass of hooked weights m (kg)	Time elapsed for 20 oscillations t (s)	Period of oscillation $T(s) = \frac{t}{20}$	Squared period of oscillation T^2 (s^2)

1) Use Excel to plot your data from Table 1 (T^2 vs m), then find your slope and y-intercept. From there, calculate your stiffness constant k and effective mass fm_s of your spring. (Show your calculations.) [10 points]

TA Signature/Initials: ________________________

Part 2

IDEA TO REMEMBER!

Springs have a constant stiffness, called *k*!

Initial length: ____________(m) [1 point]

Table 2: Force vs Displacement [8 points]

Mass of hooked weights m (kg)	Displacement from compressed to hanging equilibrium y (m)	Weight due to *m* F (N)

2) Using an Excel Scatter graph and Trendline, plot *F* vs *Δy* from Table 2 to find the slope. Why did we plot *F* vs *Δy*? [10 points]

TA Signature/Initials: ____________________

3) How different was your result between the two methods? Which calculation for the spring constant would you believe to be more reliable, and why? [5 point]

Part 3

IDEA TO REMEMBER!

Springs have a constant stiffness, called *k*!

4) Given the harmonic motion position equation *y(t)* and your data from Part 2, answer the following: [10 points]

a) What should amplitude *A* be? Show your work.

b) Assuming you know the spring stiffness *k* (use Part 2 data), how do you calculate ω? Show your work.

c) Assuming you do not know the spring stiffness *k* but you can trust the sensor data or you can time the period of the noise, how do you calculate ω? Show your work.

d) What should ϕ be, theoretically, since our equation uses *sin()* instead of *cos()*? If we chose to use *cos()* for our *y(t)* equation, what would ϕ be? (Hint: this is measured in radians, and depends on when the mass was released.)

5) Calculate the position of the mass at t = 1s and 1.4s using your sensor variables. Show your work. Compare your answer with the coordinate tool. [6 points]

Conclusion

Write 3 or more sentences summarizing this concept, how this lab helped you understand the concept better, and the real world implications you see. Do you still have questions? If so, write those here as well. [10 points]

IDEA TO REMEMBER!

Springs have a constant stiffness, called *k*!